Forward and Inverse EEG Source Modeling

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Source modeling

forward problem

physiological source electrical current

body tissue volume conductor

observed potential or field

inverse problem

Forward Problem

Inverse Problem

EEG or MEG

R. Oostenveld, Z. Akalin Acar & S. Makeig, 2010
Symmetry, orientation and activation

radially symmetric, i.e.
randomly-oriented

asynchronously activated

synchronously activated
parallel-oriented

R. Oostenveld & S. Makeig, 2010
Neuronal currents

Stellate cell

Closed field

Pyramidal cell

Open field

EEG

MEG

R. Oostenveld & S. Makeig, 2010
scalp dynamics ≠ source dynamics!

Cortex

Electrodes

Skin

Skull

Local Synchrony

Equivalent Current Dipole

Relative Independence

Local Synchrony
EEG volume conduction

- Potential difference between electrodes is measured. This corresponds to current flowing through skin:
  - Only tiny fraction of current passes through skull
  - Therefore the model should describe both skull and skin as accurately as possible.

- Problems with skull modeling
  - Poorly visible in anatomical MRI (T2)
  - Thickness varies
  - Conductivity is not homogeneous
  - Complex geometry at front and base of skull

R. Oostenveld 2008
Exact Formulation of the Forward Problem

$\nabla \cdot (\sigma \nabla \Phi) = -\nabla \cdot J^P \quad \text{inside} \ V$

$\sigma \frac{\partial \Phi}{\partial n} = 0 \quad \text{on} \ S$

$\sigma(x,y,z)$ : conductivity distribution
$\rho$ : current source

Z. Akalin Acar, 2010
To Solve the Forward Head Model Problem …

WE NEED

→ Head Model
  - Conductivity values
  - Geometry

→ Sensor Locations

→ Possible source distribution
  - Magnitudes
  - Locations
  - Directions

→ Solver
Source Localization Requirements

- Selected/processed EEG signal
  → Simple single-source scalp map!
- Number/positions of electrodes on the head surface
- Numerical head model
- Co-registration of EEG electrodes with head model
- A priori information/guess about the source space
- Choice of inverse model
- Choice of numerical method

Z. Akalin Acar, 2010
Volume conductor model

- Electrical properties of tissue
- Geometrical description
  - spherical model
  - realistically shaped model

→ Describes how the currents flow, from where they may originate
Errors in Simple Head Models

→ In the volume conductor model
→ In the electrode locations

Z. Akalin Acar, 2010
Head Model Comparison

- **Simple head models**
  - Single sphere
  - 3-4 Layer Spherical
  - Spheroid

- **Realistic head models**
  - Boundary Element Method
  - Finite Element Method
  - Finite Difference Method

Z. Akalin Acar, 2010
Effects of Head Model

Spherical head model
(3-layer standard)

Standard MNI head model
(4-layer mean BEM)

Z. Akalin Acar, 2010
Spherical volume conductor

• Advantages of the spherical head model
  – mathematically exact
  – fast to compute
  – reasonably accurate
  – easy to use

• Disadvantages of the spherical model
  – difficult to align properly
  – inaccurate in some regions

R. Oostenveld & S. Makeig, 2010
Realistic volume conductor

- Advantages of a realistic head model
  - a more accurate solution (especially for EEG)
- Disadvantages of a realistic model
  - more work to build from an MR image
  - slower to compute
  - might be numerically instable
  - harder to make between-subject comparisons

→ A pragmatic (easy, cheap) solution is to use a standard (mean) realistic head model (MNI).

R. Oostenveld & S. Makeig, 2010
Realistic volume conductor

• Computational methods for volume conduction problem that allow realistic geometries:
  – Boundary Element Method (BEM)
  – Finite Element Method (FEM)

• Geometrical description
  – Triangles (planar or quadratic)
  – Tetrahedra (3-D)
BEM volume conductor

- Boundary Element Method
  - describes the volume conductors
  - each compartment is
    - homogeneous
    - isotropic
  - important tissues
    - skin
    - skull
    - brain
    - (CSF)
  - triangulated surfaces
  - surfaces should
Electromagnetic source localization using realistic head models (NFT)

Zeynep Akalin Acar, ‘06
A Four-Layer BEM Head Model

Neuroelectromagnetic Forward head modeling Toolbox (NFT)

# of elements

Scalp: 6900
Skull: 6800
CSF: 9000
Brain: 8800

Total 31500

Z. Akalin Acar, 2010
FEM volume conductor

• Tesselate the 3-D volume into solid tetrahedra
  - Large number of elements
  - Each tetrahedron can have its own conductivity
  - Each tetrahedron can have its own anisotropy

• FEM is most accurate numerical method
  – Computationally expensive
  – Accurate conductivities are not known

R. Oostenveld & S. Makeig, 2010
Inverse problem methods

• **Single and multiple dipole models**
  – Minimize error between the model and the measured potential/field

• **Distributed dipole models**
  – Perfect fit of model to the measured potential/field
  – Minimize an additional constraint on sources
    • LORETA (assume a smooth distribution)
    • Minimum Norm (L2, minimum power at the cortex)
    • Minimum Current (L1, minimum current in the cortex)
Inverse problem methods

• Spatial source filtering
  – **Scan whole brain** with single dipole and compute the filter output at every location (second-order covariance matrix)
    • MUSIC algorithm
    • *Beamforming* (e.g., LCMV, SAM, DICS)
  – **Perform ICA decomposition** (higher-order statistics)
    • Of the scalp maps at individual moments
    • ICA gives the projections of the sources to the scalp surface, i.e., ‘simple’ maps!

→ ICA solves ‘the first half’ of the inverse problem (‘What?’)
Equivalent current dipoles

• Physical/mathematical motivation
  – Any current distribution can be written as a multipole expansion
  – First term: monopole (must be 0)
  – Second term: dipole
  – Higher order terms: quadrupole, ...

• Convenience
  – **Dipoles** can be used as building blocks in distributed source models
Equivalent current dipoles
Equivalent current dipole

R. Oostenveld & S. Makeig, 2010
Measured Errors in Dipole Source Localization

- **Experimental studies**
  - Phantom $\rightarrow$ 10 mm loc. error (Henderson & Butler, 1975)
  - Human skull $\rightarrow$ 35 mm (Weinberg et al, 1986)

- **Simulation studies**
  - 3-layer model $\rightarrow$ 15-25 mm (Roth et al, 1993)
  - 3-layer model $\rightarrow$ 9-14 mm (Vanrumste et al, 2002)
  - Human skull $\rightarrow$ 25 mm (Fletcher et al, 1993)
  - 3-layer model $\rightarrow$ $\sim$8 mm (Akalin Acar, 2005)
Source Localization Errors

- For a 3-layer spherical head model
- Relative to 4-layer realistic BEM head model

Z. Akalin Acar, 2010
Single vs. multiple dipole models

• Manipulate source parameters to minimize error between measured and model data
  – **Position** of each source
  – **Orientation** of each source
  – **Strength** of each source

• **Orientation** and **strength** together correspond to the “dipole moment” and can be estimated *linearly*
  – **Position** is estimated *non-linearly* by iterative source parameter estimation
Dipole scanning: grid search

• Define grid with allowed dipole locations
• Compute optimal dipole moment for each location
• Compute value of goal-function
• Plot value of goal-function on grid
• Number of evaluations:
  – single dipole, 1 cm grid: ~4,000
  – single dipole, ½ cm grid: ~32,000
  – BUT two dipoles, 1 cm grid: ~16,000,000

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Dipole fitting: nonlinear search

• Start with an initial guess from coarse fitting
  – evaluate the local derivative of goal-function
  – “walk down hill” to the most optimal solution

• Number of evaluations needed ~ 100
Effect of Number of Electrodes

- Single dipole source
- 3-layer spherical head model
- 1152 solution points

Michel et al, 2004

Z. Akalin Acar, 2010
Effects of Skull Conductivity Estimate

Measurements of skull conductivity:

- In vivo
  - MR-EIT
  - Magnetic stimulation
  - Current injection

- In vitro

Hoekama et al, 2003

He et al, 2005

Z. Akalin Acar, 2010
### Effects of Skull Conductivity Estimate

#### Brain to skull ratio

<table>
<thead>
<tr>
<th>Study</th>
<th>Year</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rush and Driscoll</td>
<td>1968</td>
<td>80</td>
</tr>
<tr>
<td>Cohen and Cuffin</td>
<td>1983</td>
<td>80</td>
</tr>
<tr>
<td>Oostendorp et al</td>
<td>2000</td>
<td>15</td>
</tr>
<tr>
<td>Lai et al</td>
<td>2005</td>
<td>25</td>
</tr>
</tbody>
</table>

#### Skull conductivity by age

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Age</th>
<th>$\sigma$ (mS/m)</th>
<th>Sd (mS/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agar-agar phantom</td>
<td>–</td>
<td>43.6</td>
<td>3.1</td>
</tr>
<tr>
<td>Patient 1</td>
<td>11</td>
<td>80.1</td>
<td>5.5</td>
</tr>
<tr>
<td>Patient 2</td>
<td>25</td>
<td>71.2</td>
<td>8.3</td>
</tr>
<tr>
<td>Patient 3</td>
<td>36</td>
<td>53.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Patient 4</td>
<td>46</td>
<td>34.4</td>
<td>2.3</td>
</tr>
<tr>
<td>Patient 5</td>
<td>50</td>
<td>32.0</td>
<td>4.5</td>
</tr>
<tr>
<td>Post mortem skull</td>
<td>68</td>
<td>21.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Source:
- Rush and Driscoll (1968)
- Cohen and Cuffin (1983)
- Hoekama et al, 2003

Z. Akalin Acar, 2010
Effect of reference electrode

“The choice of a particular reference electrode … does not change in any way the biophysical information contained in the potential distribution. It does not in any way change the relation between source and potential, except for an additive constant of no physical significance.”

- Geselowitz, 1998
Distributed source models

• Position of the source is not estimated as such
  – Pre-defined grid (3-D volume or cortical sheet)
  – Strength is estimated at each grid element
  – In principle, a linear problem, easy to solve, BUT...
    • More “unknowns” (parameters) than “knowns” (channels, measurements)
    • An infinite number of solutions can explain the data perfectly (not necessarily physiologically plausible!)
  – So, additional constraints are required ...
Conformal cortical patch source model
Conformal cortical patch source model

Model a source estimate as a sum of overlapping patches
Comparing source models
for an IC of an intracranial data set

Estimated IC cortical projection

Equivalent Current Dipole Model

Sparse Patch Basis Model

Z. Akalin Acar, 2010
Summary I

• Forward modeling is required for the interpretation of scalp topographies

• Interpretation of scalp topographies is **inverse modelling** “source estimation”

• Mathematical techniques are available to aid in interpreting scalp topographies

→ These are **inverse source models**
Summary II

• Inverse modeling
  – Model assumption for volume conductor
  – Model assumption for source (i.e., dipole)
  – Additional assumptions on source
• Single point-like sources
• Multiple point-like sources
• Distributed sources
  – Different mathematical solutions
    • Dipole fitting (linear and nonlinear)
    • Linear estimation (regularized)
End
NFT Overview

Solve the forward problem using realistic head models (BEM or FEM incorporating fibre tract anisotropy)

Mesh generation

Simple Inverse Problem

Co-registration

ICA/time-freq analysis

MRI/CT/DTI head image

Segmentation

Source Image

Solve the forward problem using realistic head models (BEM or FEM incorporating fibre tract anisotropy)

Z. Akalin Acar, 2010
Independent cortical components

- Single dipole component
- Dual-symmetric dipole component
- Equivalent dipoles

Julie Onton & S. Makeig (2006)
gyral source

sulcal source

Zeynep Akalin Acar, S. Makeig, G. Worrell, '09
“Surely, if there were gravity waves, we would have detected them by now.”
Electromagnetic source localization using realistic head models – an intracranial monitoring model

Zeynep Akalin Acar, S. Makeig, G. Worrell, ‘09
ICA in practice

Onton & Makeig, 2006
Electric field ↔ Magnetic field

Right-hand rule

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MEG volume conduction

• Measures sum of fields associated with
  – Primary currents
  – BUT also secondary currents !!!

• Only tiny fraction of current passes through the poorly conductive skull.
  – Therefore skull and skin can be neglected in the MEG model.

• Local conductivity around dipole important
  – geometry
  – conductivity

R. Oostenveld & S. Makeig, 2010
Differences between EEG and MEG

- Scalp distribution more blurred due to volume conductor in EEG
- MEG is insensitive to radial sources
- EEG sees more
- EEG more noisy in itself (electrode-skin impedance)
- MEG more sensitive to environmental noise!
- MEG requires no gel
- MEG requires the head to stay fixed!

R. Oostenveld & S. Makeig, 2010
Differences between EEG and MEG

• EEG potential differences, requires choice of reference electrode
• MEG sensors are measured independently of each other
• MEG can use simple but somewhat accurate volume conduction model
  – multiple non-concentric sphere model, i.e. each sensor has its own local sphere fitted to the
    head position of brain relative to MEG sensors
  – may vary within a long session
  – is different between sessions

R. Oostenveld & S. Makeig, 2010
Motivation

• Why fit dipoles?
• Why measure EEG?
• Why do ICA?
• Get extra information about brain processes
  – Time course of activity ----> EEG
  – Location of activity → fMRI

R. Oostenveld & S. Makeig, 2010
Differences between EEG and fMRI

• EEG measures post-synaptic potentials
  – related to synchronized neuronal input (phase)
• fMRI measures BOLD
  – related to energy consumption (amplitude)
• Different characteristics in the time domain
• Different generators
• Time course

R. Oostenveld & S. Makeig, 2010
Why EEG?: extra information

• Timecourse
  – ERSP
  – ERP

• Topography
  – Scalp distribution
  – Underlying
The Forward & Inverse Problems

Forward Problem

Inverse Problem

EEG/MEG
Effect of skull conductivity

Head model: 4-layer MR-based BEM model,
brain to skull conductivity ratio = 40
Reference head model -> ratio = 80
The EEG Forward / Inverse Problem

Zeynep Akalin Acar
Swartz Center for Computational Neuroscience
University of California San Diego
June, 2010
Source Localization Error

• For a 3-layer spherical head model
• Relative to 4-layer realistic BEM head model
Source Localization Error

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Dipole Orientation Error

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Dipole Orientation Error

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Z. Akalin Acar, 2010
Source Localization Error

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Z. Akalin Acar, 2010
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Z. Akalin Acar, 2010
Dipole Orientation Error

- For the **standard MNI** head model
- Relative to 4-layer realistic BEM head model
Effect of Electrode Co-registration Error

Electrode locations tilted 5 degrees backwards
Effect of Electrode Co-registration Error

Electrode locations tilted 5 degrees leftwards
Effect of Skull Conductivity Estimate

Head model: MR-based 4-layer BEM,
Brain-to-skull conductivity ratio: 60 versus 80
Effect of Skull Conductivity Estimate

Head model: MR-based 4-layer BEM,
Brain-to-skull conductivity ratio: 60 versus 80
Effect of Skull Conductivity Estimate

Head model: MR-based 4-layer BEM,
Brain-to-skull conductivity ratio: 60 versus 80
Effect of Skull Conductivity Estimate

Head model: MR-based 4-layer BEM, Brain-to-skull conductivity ratio: 60 versus 80
Effect of Skull Conductivity Estimate

Head model: MR-based 4-layer BEM, Brain-to-skull conductivity ratio: 40 versus 80
Effect of Skull Conductivity Estimate

Head model: MR-based 4-layer BEM,
Brain-to-skull conductivity:

ratio=60, max $\Delta = 7$ mm
ratio=40, max $\Delta = 15$ mm
ratio=20, max $\Delta = 27$ mm
Localization Error for Patch Sources

- Source space: Multiscale conformal cortical patches
- Forward model: MR-based 4-layer BEM model
- Inverse model: Equivalent current dipole
Localization Error for Patch Sources

Head model: MR-based 4-layer BEM

Patch size = 10 mm  Patch size = 6 mm  Patch size = 3 mm
Localization Error for Patch Sources

Head model: 4-layer MR-based BEM

Patch size = 10 mm  Patch size = 6 mm  Patch size = 3 mm

Z. Akalin Acar, 2010
Localization Error for Patch Sources

Head model: Standard MNI

Patch size = 10 mm  Patch size = 6 mm  Patch size = 3 mm
Localization Error for Patch Sources

Head model: 3-layer spherical

Patch size = 10 mm  Patch size = 6 mm  Patch size = 3 mm

Z. Akalin Acar, 2010